Working Paper No. 102

DESCRIPTIVE AND NORMATIVE IMPLICATIONS OF THE DECISION THEORY POSTULATES

by

KENNETH R. MacCRIMMON

March-April, 1966

WESTERN MANAGEMENT SCIENCE INSTITUTE
University of California, Los Angeles

DESCRIPTIVE AND NORMATIVE IMPLICATIONS OF THE DECISION THEORY POSTULATES

A Summary of Experimental Results with Business Executives

by

Kenneth R. MacCrimmon
Carnegie Institute of Technology

Carnegie Institute of Technology
Ford Project in Organizational Behavior
Working Paper No. 21R

and

University of California, Los Angeles
Western Management Science Institute
Working Paper No. 102

A paper prepared for the
International Economic Association Conference
on "Risk and Uncertainty"
Smolenice Castle, Czechoslovakia
March 30-April 5, 1966

ACKNOWLEDGMENTS

I gratefully acknowledge my great debt to Jacob Marschak. His support, guidance, suggestions, encouragement and criticisms made possible the study underlying this paper. The general assistance of David Hitchcock was an important factor in the study. I wish to thank Leo Breiman, James Jackson, and Anthony Oberschall for their valuable suggestions and criticisms, and to acknowledge the helpful discussions with, or assistance of, George Brown, Robert Hagin, James MacQueen, Ross Nelson, John Torell, Jerome Wiest, and Robert Wilson. The cooperation of Joseph Carrabino, the staff of the UCLA Executive Program, and Program Groups 19 and 20 is particularly appreciated. Most of the research reported in this paper, and the preparation of the paper itself, was supported by the Ford Foundation in the form of fellowships and by a grant to the Organizational Behavior Project (1-40055) at Carnegie Institute of Technology. The responsibility for shortcomings and conclusions is, of course, solely the author's, and cannot be attributed to any of the individuals or groups named above.

ABSTRACT

This study deals with decision making under uncertainty, and more specifically with a set of postulates (based on the concepts of utility and personal probability) for consistent decision making. It explores the degree to which these postulates are good descriptions of actual behavior and the degree to which they seem to be good norms. Experiments in the form of decision problems based on each of the postulates were given to experienced business executives. In addition to obtaining basic responses to these problems, a systematic follow-up was made both in written and later in oral form in order to probe the normative implications of the postulates.

We conclude, based on the basic responses, that the decision making theory under study is not a very good description. However--and this is the important point--the subjects tend to regard most of the deviations from the theory as mistakes and if given the opportunity will correct them. This suggests the importance of the postulates in training decision makers in both how to structure decision problems and also how to police their own decisions.

1. INTRODUCTION

Most decisions of importance are made under conditions of uncertainty. In choosing among various courses of action we are not sure what the outcome of any particular action will be. A number of decision making theories have been proposed for such situations. In this paper we shall deal with one particular theory of decision making under uncertainty. This theory is based on the concepts of personal probability and utility and may be stated in the form of a set of postulates. Various sets of postulates have been formulated; the postulates we shall use are essentially the same as those of L. J. Savage [1954].

This decision theory is normative rather than descriptive. That is, it is a theory of how individuals <u>should</u> make decisions rather than how they <u>do</u> make decisions. Several challenges to the normative implications of the theory have been made in the form of conjectures and "paradoxes." These have generally been unsupported by empirical evidence. On the other hand, some researchers have supposed the theory to be descriptive and have generated much data. Unfortunately, many of these studies do not have the presumed relevance to the theory. In this study we attempt to discover the degree to which the postulates are good descriptions (of actual behavior) and the degree to which they seem to be good norms.

A descriptive theory can be judged by its explanatory or predictive ability. It is more difficult, though, to judge a normative theory. Presumably adopting a good normative theory will lead to "better" results. But "better" in what sense? The criteria must be specified and will often be part of the theory itself. One condition we might expect a good normative theory to satisfy is that it should seem reasonable to individuals with expertise in the domain of usage. Thus, we would expect a good normative theory of decision to seem reasonable to successful, practicing decision makers.

To this end we present some experimental results based on decision problems given to a group of business executives. These executives were drawn from the Executive Development Program at the University of California, Los Angeles. This program trains upper-middle level executives for top management positions. The average* executive can be described as 42 years of age, holding a college degree, earning $25,000/year as a division manager in a company of about 6,000 employees.

In a series of four experimental sessions (of about one hour each) held in March and April 1964, decision problems based on each of the postulates were given to the subjects. Thirty-eight executives became regular subjects, and with only a couple of minor exceptions completed all the material. In this paper we can only briefly summarize the

---

* The figures are based on median values for each characteristic.

experiments and the results. For completeness we shall mention the results for each postulate. We shall, however, particularly emphasize those experiments and results having the greatest relevance to the topic of uncertainty. For a more complete treatment the reader is referred to MacCrimmon [1965] and a series of forthcoming working papers.

In order to probe the normative implications of the postulates we were not only concerned with the subjects' initial response to the decision problems, but we were interested in their behavior when they were given an opportunity to reflect on their answers—and especially after they were presented with arguments conforming to, and conflicting with, the postulates.* For this reason, the decision problems were specially constructed so that the strongest possible counter-argument to each postulate could be made. This puts the postulates to a very severe test, perhaps more severe than would arise in most real-world decision situations.

Although it is desirable to present as much experimental material as possible in a written form (to allow replication by other researchers), it is very important in experiments such as these to have an opportunity to follow-up on interesting or confusing written responses. Thus, after all the written experimental material had been completed, a terminal interview (of median length, 30 minutes) was

---

* These arguments were presented in a written form and will be described in more detail in later sections of this paper.

held with each subject individually. These interviews are an important part of this study, but because of constraints on the length of this paper it is feasible to present only a small part of the interview results. The interested reader is referred to the reports mentioned above.

In the next six sections of this paper, we shall state the appropriate postulate, present one of the experiments based (primarily) on that postulate, describe the experimental procedures, and discuss the results of the relevant experiments. Before presenting this material, however, we need to introduce the following basic concepts.

Alternatives (also acts or actions):

$a, b, c \in A$, the set of all possible actions

States:

$x \in X$, the set of all possible states of the world

Events:

$z \subseteq X$

If $\bigcup_{i=1}^{n} z_i = X$ and $z_i \cap z_j = \emptyset$ (all i, j, $i \neq j$),

we call $\{z_i\}$ an n-fold partition of X.

$z^c = \{x \in X : x \notin z\}$, and is called the complement of z

Consequences (also outcomes or rewards):

$(a,x) \in A \times X$

If (a,x) is a constant for all $x \in z$, we write (a,z).

These concepts plus the relation of preference provide the basis for the postulates. The preference relation is defined as follows: if the decision maker chooses b when both a and b are available, we say that a "is not preferred to" b , and write $a \preccurlyeq b$. As a corollary, if $a \preccurlyeq b$ and also $b \preccurlyeq a$, we say that the decision maker "is indifferent between" a and b , and write $a \sim b$; if $a \preccurlyeq b$ but it is not true that $b \preccurlyeq a$, we say that b "is strictly preferred to" a , and write $a \prec b$. We later (Postulate 4) use the preference relation to develop the relation "is not more probable than" between events. This relation is denoted as $\leq_{\cdot}$.

2. POSTULATE 1: COMPLETE ORDERING

The first postulate we consider asserts that all alternatives are comparable and that choices are transitive. A formal statement of the postulate is given below.

Postulate 1: For every $a, b, c \in A$

(1) $a \preccurlyeq b$ or $b \preccurlyeq a$

(2) if $a \preccurlyeq b$ and if $b \preccurlyeq c$, then $a \preccurlyeq c$.

The first part of the postulate (comparability) was not an explicit part of our experiments. We note though that at no point did a subject refuse to make a choice. The second part of the postulate (transitivity) was the basis for a particular set of

experiments. These experiments will be treated only briefly in this paper because although transitivity is a very important concept, it is not a notion especially concerned with the topic of uncertainty.

As an example of the experiments used, the context of the first experiment was as follows:

> Assume you are President of a company that makes small appliances. Suppose your company has extensively redesigned and improved one item in your product line. You must now make the final decision on the new price.
>
> You feel that the important criteria in evaluating a pricing policy are expected return and expected share of market. You have directed your market research staff to prepare these figures for the various possible pricing policies. You have every reason to believe in the reliability of the figures.

The subject was asked:

> Which of the following pricing policies, A or B, do you prefer? Please circle your choice.
>
> A) Expected return: 10%
> Expected share of market: 40%
>
> B) Expected return: 20%
> Expected share of market: 20%

A total of 4 distinct alternatives were used (the others being C) Expected return: 5%, Expected share of market: 50%, and D) Expected return: 15%, Expected share of market: 30%). All 6 binary combinations of these 4 alternatives were presented (interleaved with some other transitivity experiments). This structure means that the subject had 4 chances to make 2 intransitivities on this experiment. [May, 1954]

After being presented these binary choices the subject was asked to rank the four alternatives. Thus in addition to observing intransitivities (i.e., circular triples) we can compare his binary choices with his ranking of the four alternatives. A discrepancy between the binary choice and the ranking will be called a "choice instability".*

In the experiment described, only 2 of the 38 subjects had an intransitivity while 18 of the subjects had "choice instabilities". In the other two transitivity experiments (with a similar structure to the one described) one subject in the second experiment and 5 subjects in the third had intransitivities, while 15 and 11 subjects, respectively, had "choice instabilities" in these experiments. No subject had an intransitivity in more than one experiment. Thus, of the 114 subject-experiments (38 × 3), 8 subjects had intransitivities.

During the interview, 6 of these 8 subjects quickly acknowledged that they had made a "mistake" in their choices and expressed a desire to change their choices. The remaining two subjects persisted in the intransitivity asserting that they saw no need to change their choices because they focused on different dimensions for different choice pairs. The fallacy of this reasoning was not apparent to them in a five-minute discussion.

---

* An intransitivity implies a "choice instability", but the converse is not true.

Only 8 of the 38 subjects had no "choice instabilities" in the three experiments. The other 30 subjects (except for the 2 that persisted in intransitivities) all wished to change their choice (most would change the binary choice, but the difference was not significant). They generally attributed the "choice instability" to carelessness in their reading or thinking.

3. POSTULATE 2: (a) THE IRRELEVANCE OF IDENTICAL OUTCOMES

Two implications of Postulate 2 will be discussed in this paper. The first, which may be called "the irrelevance of identical outcomes" is the subject of this section, and the second implication will be covered in the next section (4). Postulate 2 is formally stated below.

> POSTULATE 2: If $(a,x) = (a',x)$ and $(b,x) = (b',x)$ for every $x \varepsilon z$, and if $(a,x) = (b,x)$ and $(a',x) = (b',x)$ for every $x \varepsilon z^c$, then $a \preccurlyeq b$ if, and only if, $a' \preccurlyeq b'$.

Three experiments concerning the "irrelevance" implication of Postulate 2 were given to the subjects. The first experiment read as follows.*

> Assume you are the President of a medium-sized company, and you have to choose among alternative investments in a production process involving uncertain outcomes. Suppose

---

* Slight changes have been made in notation and format, so the current reader can more easily relate the experiment and Postulate 2.

further that the uncertainty can be related to drawing a ball out of an urn containing 100 balls numbered from 1 to 100. The ball drawn will determine the outcome.

Which of the following two investments do you choose? (Please give reasons for your choice.)

Investment a: Invest in a production process such that if ball numbered 1-10 is drawn, it corresponds to getting a return of 500%; if ball number 11 is drawn, it corresponds to failure of the process and your company will go bankrupt; and finally if ball numbered 12-100 is drawn, it corresponds to getting a return of 5%.

Investment b: Invest in a production process such that no matter which ball (numbered 1-100) is drawn, it corresponds to getting a return of 5%.

After making a choice between a and b, the subject was then asked:

Making the same assumptions as above, if instead of investments a and b you are faced with investments a' and b', which would you choose? (Please give reasons for your choice.)

Investment a': Invest in a production process such that if ball numbered 1-10 is drawn, it corresponds to getting a return of 500%; however, if ball numbered 11-100 is drawn, your company will go bankrupt.

Investment b': Invest in a production process, su[illegible] that if ball numbered 1-11 is drawn, it corresponds to getting a return of 5%; however, if ball numbered 12-100 is drawn, your company will go bankrupt.

For the reader's convenience this experiment is shown in the payoff matrix below.

	Balls 1-10	Ball 11	Balls 12-100
Investment a	500% return	bankruptcy	5% return
Investment b	5% return	5% return	5% return
Investment a'	500% return	bankruptcy	bankruptcy
Investment b'	5% return	5% return	bankruptcy

Postulate 2 asserts that a choice of b over a should lead to a choice of b' over a' , and conversely. (Note that event z corresponds to Balls 1-11, and event z^c corresponds to Balls 12-100.)

This type of problem has been suggested by M. Allais [1953, 1953a] and G. Morlat [1953]. They conjecture that individuals when confronted with such situations will choose alternative b over a , and alternative a' over b' . The assertion is made [Allais, 1953] that even upon reflection individuals will maintain such a choice. Persistence in such a choice (or in a and b') is, of course, in conflict with Postulate 2.

In our experiments, after answering the above questions the subject was presented with two prepared answers to the same problem. He was told both answers had been given by executives in a previous session. He was asked to provide a critique of each answer and to select the one that was more logical.

One prepared answer conformed with Postulate 2. It chose alternatives b and b' and gave reasons to the effect that the outcomes for the pair a and b , and for the pair a' and b' , were identical if a ball numbered 12 through 100 was drawn, and, on the basis of taste, b was preferred to a when considering balls 1-11, but since b' is identical to b , and a' is identical to a , this implied b' should then be chosen over a' .

The other prepared answer had choices that conflicted with Postulate 2. The reasons presented were similar to those suggested by Allais [1953]. The choices made were b and a' . The justification for b was that it was safer--i.e., why take a chance on bankruptcy--whereas the justification for a' was that both a' and b' involved a high (and almost indistinguishable) chance of bankruptcy and so one might as well aspire to a 500% return.

In the decision problem described above (the first of three quite similar experiments), slightly over 60% of the (36 participating) subjects gave initial answers consistent with Postulate 2. Thus, almost 40% had answers in conflict with Postulate 2. These same percentages of conformity-conflict responses were found in the other two similar experiments. (The second experiment changed some of the material to a qualitative form, and the third experiment reversed the ball numbers and narrowed the quantitative outcomes.) A high degree of apparent conflict with Postulate 2 thus exists in the actual responses of the subjects.

Information on the extent to which the figures represent accidental conformity or accidental violation may be obtained from the (written) responses to the prepared answers. On the first experiment, only about 20% of the subjects selected the conforming answer, while about 80% selected the conflicting answer as being the more logical one. Note then that many individuals who themselves gave conforming answers disagreed with the prepared conforming answer. (It is also true that some subjects initially giving violating answers disagreed with the prepared violating answer.) These percentages shifted toward conformity on the second experiment, with about 50% of the subjects selecting the conforming answer and about 50% of the subjects selecting the violating answer. (There was no presentation of prepared answers in the third experiment.)

Some logical lapses on the part of the subjects are thus apparent. Even though an individual himself may conform or violate Postulate 2, he apparently cannot recognize a complementary conforming or violating answer. Perhaps the prime reason for this response pattern was uncovered during the interview. The subjects had great difficulty separating logical deductions from a set of premises (the operation that was requested here) from the beliefs underlying the premises themselves. Thus, for example, if an individual selected a and a' (a conforming answer) he may select the violating prepared answer of b and a' because the prepared conforming answer, b and b' , is composed of two completely different actions. Reasons given

for this choice would often be that the (b,b') prepared answer was "too conservative," rather than asserting it was less logical than the other prepared answer. (Analogous responses were given in the other cases.) Reasons given for the prepared answer choice often had no connection with the subject's own answers and reasons given just a few minutes earlier.

The oral interview provided an opportunity to clear up some of this confusion between beliefs and logical deductions. After such a discussion most of the subjects tended to move toward conformity with the postulate-based reasoning. This trend can even be noted in the strictly written part of the experiment by comparing the figures given above for the responses to the prepared answers in the first and second experiments. The shift became more pronounced during the interview with about 75% of the subjects indicating their complete agreement with the postulate-based reasoning. Another 11% indicated agreement for all instances except where one alternative has a sure outcome (for example, in the experiment described earlier, alternative b always gives a 5% return).

Approximately 14% of the subjects indicated their continued disagreement with the applicability of the postulate-based reasoning to such decision problems. The reason most often given by these subjects was that the problem could not be decomposed as the postulate-based reason implied. Comments such as "you can't disregard what happens if balls 12-100 are drawn," "you can't ignore such a big part of the

problem," and "you're only looking at part of the picture," were typical of this group. Attempts to explain that events were not being "ignored" were unsuccessful. It should be observed though that all these subjects would be willing to conform to Postulate 2 if the event having identical outcomes was not so probable. Thus, that we get persistent violation of Postulate 2 is primarily due to the very special nature of these constructed situations.

4. POSTULATE 2: (b) RISK versus UNCERTAINTY

In this section we shall consider another implication of Postulate 2 (formally stated on p. 8). This interpretation may be called "risk versus uncertainty." We can develop the meaning of this most usefully in the context of one of the three experiments actually given to the subjects. The core of the first experiment read as follows:

1. Which of the following wagers do you prefer?

   (a) $1000 if the top card in an ordinary, well-shuffled deck of playing cards is red; $0 if black.
   (b) $1000 if the closing price tomorrow of Pierce Industries on the American Stock Exchange is higher than the closing price today; $0 if not higher.
   (c) Either (a) or (b)

   Please give reasons for your choice.

2. Which of the following wagers do you prefer?

   (a) $1000 if the top card in an ordinary, well-shuffled deck of playing cards is black; $0 if red.
   (b) $1000 if the closing price tomorrow of Pierce Industries on the American Stock Exchange is not higher than the closing price today; $0 if higher.
   (c) Either (a) or (b).

   Please give reasons for your choice.

This abbreviated form of the first experiment can serve to demonstrate the main focus of our interest in this section, that is, a "risk/uncertainty" implication of Postulate 2.

The applicability of Postulate 2 to this experiment can perhaps best be seen by considering the payoff matrix representation below.

Actions \ Events	Stock higher & Card black	Stock not higher & Card red	Stock higher & Card red	Stock not higher & Card black
1(a)	$0	$1000	$1000	$0
1(b)	$1000	$0	$1000	$0
2(b)	$0	$1000	$0	$1000
2(a)	$1000	$0	$0	$1000

Now making a correspondence with the notation of Postulate 2 (p. 8), let us associate the (joint) events "stock higher & card black" and "stock not higher & card red" with z, the (joint) events "stock higher & card red" and "stock not higher & card black" with z^c. Let us further

associate the actions 1(a) with a , 1(b) with b , 2(b) with a' , and 2(a) with b' . For convenience in the following discussion let us refer to 1(a) as a "bet" on the card being red, 1(b) as a "bet" on the stock price being higher, 2(b) as a "bet" on the stock price not being higher, and 2(a) as a "bet" on the card being black.*

Using these correspondences, Postulate 2 asserts that if you prefer a bet on the card being red over a bet on the stock price being higher, you then should prefer a bet on the stock price not being higher, than a bet on the card being black. Note that the event "black" is the complement of the event "red", and the event "not higher" is the complement of the event "higher". Postulate 2 thus asserts that if you prefer a bet on one event over a bet on a second event, then you should prefer a bet on the complement of the second event over a bet on the complement of the first event (given the same reward conditions). This relationship holds regardless of the nature of the events. Thus, within the context of this theory it is not meaningful to distinguish between the nature of events.

A notion that is in conflict with this is the dichotomy between "risky events" and "uncertain events" that is sometimes made. Under such a view, the card events would be called "risky" because most people would be willing to assign a (common) relative frequency to them, while the stock events would be called "uncertain" because most people would not

---

* By a bet on an event we mean an alternative such that the decision maker gains more if that event occurs than he does if it does not occur.

assign (common) relative frequencies. This leads to an incomparability between "risky events" and "uncertain events" and to a different procedure for dealing with them. Thus, with such a view, it would not be inconsistent to prefer the bet on "red" to the bet on "higher" and then to prefer the bet on "black" to the bet on "not higher".

This conclusion, and the type of experiment itself, has been suggested by D. Ellsberg [1961] as a situation in which people do not act in a manner consistent with Postulate 2. He contends that on the basis of presenting similar, but informal, problems to some top decision theorists, these individuals generally do choose actions 1(a) and 2(a), i.e., the bets on the "risky events" over the bets on the "uncertain events". Ellsberg further claims that even upon reflection individuals will tend to persist in these choices.

In our experiments the subject was asked to make choices and give reasons in each of the six possible pairs of wagers (i.e., bets on two events and their complements taken pairwise) with the (identical) reward conditions used in the example at the beginning of this section. However, it should be noted that a choice of a bet on "red" may not indicate strict preference but only indifference. Thus, a choice of a bet on "red" and, in a later pair, a choice of a bet on "black" may not be inconsistent with Postulate 2 if they merely indicate indifference. Therefore, a second set of reward conditions was also used. Under these conditions the stock bets still paid off $1000 if correct, but the payoff on the card bets

was reduced to $990. It is easily seen that a choice of both the "red" and "black" bets under these conditions is inconsistent with Postulate 2. Altogether, then, twelve pairs of wagers were presented to the subject in a single experiment.

Three "risk/uncertainty" type experiments were given to the subjects. The first has been discussed as the example in this section. The third was very similar, the only change being a replacement of the card events ("red" or "black") by coin events ("head" or "tail") and a replacement of the stock events ("higher" or "not higher") by GNP events ("U.S. '64 GNP less than 620 billion" or "U.S. '64 GNP at least 620 billion"). The second experiment involved investments in two foreign countries with historical frequencies being given for one country (a "risky" case) while no probabilities were assigned for the other country (a case of "uncertainty").

In addition to asking for the subject's own responses on the first experiment, two different sets of choices and reasons were presented (in written form) to him for his critique. One set was consistent with Postulate 2, using informal probability type arguments, while the other was inconsistent with Postulate 2 in that it gave "risk/uncertainty" type arguments--stating that the card odds were known whereas the stock odds were not known. Both sets of written reasons were entirely verbal in form.

Thirty-five of the 38 subjects completed the three experiments and the corresponding part of the terminal interview. Various kinds of answers inconsistent with Postulate 2 can be identified [MacCrimmon, 1965], but here we shall only be concerned with the "risk/uncertainty" ones. (Most of the others were later labeled as "mistakes" by the subjects.)

In the first experiment (the example at the beginning of this section) only 3 subjects had "risk/uncertainty" violations of Postulate 2, in the third experiment only 5 subjects had such violations, while in the second experiment 19 subjects had "risk/uncertainty" violations. Later questioning of the subjects indicated that while the second experiment provided a less artificial decision context it afforded an opportunity to make many irrelevant and changing assumptions that led to an unusually high degree of conflict with Postulate 2.

Considering all three experiments (even though the second has the defects mentioned above), no subjects had three "risk/uncertainty" violations, only 3 subjects had two "risk/uncertainty" violations, and 11 subjects had no "risk/uncertainty" violations. The remaining 21 subjects had one "risk/uncertainty" violation, with 17 of these occurring on the second experiment. If we separately consider only the first and third experiments (the two most closely corresponding to Ellsberg's), we find that 28 subjects had no "risk/uncertainty" violations, 6 subjects had one, while only 1 subject had a "risk/uncertainty" violation in both experiments.

In the first experiment a relatively higher proportion agreed with the violating answer than gave violating answers themselves; however, a majority still thought that the conforming answer was more logical. Eighteen subjects selected the conforming answer, 11 subjects selected the "risk/uncertainty" violating answer, while the other 6 subjects thought neither was logical. Contrary to the responses to the prepared answers for the experiments discussed in the previous section (section 3), there was an insignificant amount of "cross-over" (i.e., subjects giving one response themselves, then later selecting the other).

At the terminal interview, the reasons for the subject's particular choices were pursued with emphasis being placed on those seemingly distinguishing between "risk" and "uncertainty". The only subject giving "risk/uncertainty" choices in his written answers to the first and third experiments persisted in this distinction during the interview. He insisted that he knew the odds on card and coin bets but not on bets involving stock price increases or the level of GNP; and further he stated that he would select bets in which he knew the odds--even at a slight penalty (in terms of the payoffs). Of the other 6 subjects having (single) "risk/uncertainty" violations in either the first and third experiment, all but one (with a second subject unsure) concluded that such a distinction was not reasonable, and they indicated a wish to change their choices. However, in the second experiment, slightly over half (10 of 19 subjects) did not wish to change a "risk/uncertainty" violation. In general, this may be related to ambiguity in the problem

statement, clarification of which the subjects would not accept. Of the other subjects--those initially giving non "risk/uncertainty" violating answers, almost all attributed their answers to "mistakes" and agreed completely with the applicability of Postulate 2.

In general, then, subjects do not persist in making distinctions between "risky" and "uncertain" events when given an opportunity to reflect on their answers--and this in an experiment especially designed to elicit such violations! Persistent violators are probably rare, but one can agree with Ellsberg [1963] that they are a particularly interesting group to study further.

5. POSTULATE 3: ADMISSIBILITY

Postulate 3* introduces a very important concept, admissibility (also called "dominance"). This notion has general interest beyond a particular theory of decision such as that considered here. This postulate is formally stated as follows:

> POSTULATE 3: If $\{z_i\}$ is an n-fold partition of z, and if $(a, z_i) \preccurlyeq (b, z_i)$ for all $z_i \in \{z_i\}$, $i = 1, 2, \ldots, n$, then $a \preccurlyeq b$ given z. If, in addition, $(a, z_{i_o}) \prec (b, z_{i_o})$ for some non-null $z_{i_o} \in \{z_i\}$, then $a \prec b$ given z.

---

* Corresponding to Savage's Theorem 3 [1954, p. 26].

When the above conditions hold we say that alternative a is "inadmissible", or that alternative b "dominates" alternative a .

The concept of admissibility appeals very strongly to one's common sense. It also can have great practical significance in decision problems. It gives us a way of choosing among complex actions in particular situations after having established a preference among consequences.

As part of the experiments on the independence of beliefs and tastes (to be discussed in the next section), material dealing with admissibility was included. In these experiments the subjects were asked to rank a number of alternative wagers from the one they preferred most to the one they least preferred. Of the six pages of wagers only the last three are of relevance here; they each contained 12 wagers.

Three pairs of wagers form the test of admissibility.

a_1: Win \$1200 if the Gross National Product for 1964 is over \$610 billion; lose \$400 otherwise.
b_1: Win \$1200 if the Gross National Product for 1964 is over \$600 billion; lose \$400 otherwise.

a_2: Win \$1200 if Goldwater receives the Republican nomination for President in 1964; lose \$400 otherwise.
b_2: Win \$1200 if Goldwater or Rockefeller receives the Republican nomination for President in 1964; lose \$400 otherwise.

a_3: Win \$1200 if the price of Syntex on the American Stock Exchange is at least twice its current price by the end of this year; lose \$400 otherwise.
b_3: Win \$1200 if the price of Syntex on the American Stock Exchange is higher than its current price by the end of this year; lose \$400 otherwise.

On each of the three pages, two of these pairs appeared; thus, each pair was presented twice. Different reward conditions were used on each page.

The applicability of the postulate of admissibility to the above pairs should be quite apparent. In each case the b alternative dominates the a alternative (i.e., $a_i \leqslant b_i$ for i = 1, 2, 3)*. Ranking the a alternative higher than the b alternative is, thus, a violation of the admissibility postulate.

Thirty-seven of the 38 regular subjects completed the admissibility experiment and participated in the interview. Of the 37 subjects, 12 had at least one inadmissibility, and 5 of the 12 had multiple inadmissibilities. Four of the 5 had two inadmissible choices, while the other subject had four inadmissibilities. (The maximum number of inadmissibilities possible, of course, was six, that is, three pairs of wagers presented twice.)

Because convincing written counter-arguments to the concept of admissibility could not be constructed, the subjects were not forced to reflect on their answers until the interview. After giving the subjects such an opportunity during the interview, almost all subjects with inadmissibilities quickly indicated that they would revise them. They made comments such as, "That's just a mistake", "I misread it", and "That other one is obviously better".

Two subjects did not recognize the inadmissibility immediately and they were questioned further about their choices (e.g., questions such as "Do you know what happens if GNP is between 600 and 610?"). These subjects then perceived the inadmissibility. One subject stated that "It

---

* Assuming a larger money amount is preferred to a smaller amount.

seems obvious now, . . . but I didn't look at it that way". The other subject said, "I don't know what I could have been thinking of".

Thus no subject persisted in an inadmissible choice. Those subjects initially giving inadmissible choices generally attributed them to carelessness in their reading or thinking about the problem.

6. POSTULATE 4: INDEPENDENCE OF BELIEFS AND TASTES

The postulate we deal with in this section asserts that the decision maker's beliefs should be independent of his tastes. He should be subject neither to wishful thinking nor to persecution mania. Postulate 4 may be formally stated as follows.

POSTULATE 4: For every $a, b \in A$ and $z_1, z_1^c, z_2, z_2^c \subset X$,
if $(a, z_1^c) \sim (b, z_2^c) \prec (a, z_1) \sim (b, z_2)$, then
$z_1 \leq^{\bullet} z_2$ if, and only if, $a \preccurlyeq b$.

The alternatives a and b may be thought of as "bets" on events z_1 and z_2, respectively, in the same sense as the term was used in section 4.

Subjects were asked to rank, from most preferred to least preferred, all bets on a page containing either 8 or 12 bets. There were 6 such pages. The bets were grouped into quadruples and various combinations of quadruples were presented in a random order on a page. Each <u>bet</u> on the same page had identical reward conditions, but each <u>page</u> had a different set

of rewards. The reward conditions comprised all possible combinations of losses and gains, and positive and negative expected money values.

An example of the type of quadruple and reward condition used is given below.

(a) Win $1200 if a fair coin falls head up; lose $400 otherwise.
(b) Win $1200 if the gold outflow exceeds the gold inflow in the U.S. this year; lose $400 otherwise.
(c) Win $1200 if Britain enters the Common Market this year; lose $400 otherwise.
(d) Win $1200 if the price of the stock Syntex on the American Stock Exchange at the end of this year will be at least twice its current price; lose $400 otherwise.

A total of four such quadruples were used.

Postulate 4 asserts that these bets should be ranked in the same order irrespective of the particular reward conditions (so long as the first is preferred to the second--for example, when win $1200, lose $400 are replaced by win $0, lose $300, respectively). Thus, on each page that a quadruple appears it should be ranked in identical order. The bets in a given quadruple always appeared mixed in with those of one or two other quadruples (the quadruples combined differed from page to page). Subjects were not allowed to look back or to keep notes.

Thirty-seven subjects completed this experiment. Only two subjects had as many as 2 of the 4 quadruples ranked identically over the 3 or 4 times the quadruple was presented. Sixteen subjects had none of the 4 quadruples ranked identically. This is an unexpectedly

high degree of apparent disagreement with the independence of tastes and beliefs postulate. That is, apparently most subjects do let their tastes influence their beliefs.

However, during the interview there was a unanimous swing toward identical rankings (by each subject for the quadruples presented). None of the 37 subjects wished to maintain even one different ranking--some even said specifically that "the probabilities don't change". The violation was primarily attributed to mistakes they had made and the complexity of the experiment. As one executive said, "I mainly consider at most 3 or 4 alternatives at a time, not 8 or 12!" There was complete agreement (after the interview) on the reasonableness of the implications of Postulate 4.

7. POSTULATE 5: EQUIVALENT N-FOLD PARTITION

The fifth postulate* is primarily a structural one. It asserts that the decision maker should be able to construct an n-fold equivalent partition of the universal state. The meaning of this is implicitly defined in the following formal statement of the postulate.

POSTULATE 5: For any n, there exists an n-fold partition $z = \{z_1, z_2, \ldots, z_n\}$ of X such that for some $a_1, a_2, \ldots, a_n \in A$, 1) $(a_i, z_i^c) = s \prec t = (a_i, z_i)$ and 2) $a_1 \sim a_2 \sim \ldots \sim a_n$.

---

* Corresponding to Savage's Postulate 6.

In the experiments we deal with only a special case of Postulate 5, the case where $n = 2$. Thus, we are examining only the subjects' ability to make an equivalent binary partition. Consider the following series of wagers.

Please indicate your choice among the following sets of wagers:

(a) Win \$1000 if the Dow Jones industrial average (of stocks on the New York Stock Exchange) at the end of this year is at least x; win \$0 otherwise.
(b) Win \$1000 if the Dow Jones industrial average at the end of this year is less than x; win \$0 otherwise.
(c) Neither or either (please explain).

The variable x takes on the values 600, 1000, 800, 700, 900, 650, 950, 750, and 850 in that order. The value of the Dow Jones industrial average at the time of the experiments was around 800.

Consistent responses would select the "at least" bet for x less than some value, x_o, and would switch to the "less than" bet for x greater than x_o. This experiment roughly indicates a zeroing-in procedure for obtaining equivalent partitions; however, note that we deal only with the special case, $n = 2$, and only coarse intervals are used.

Of the 37 subjects who completed this experiment, 32 gave choices (of the above form) from which a unique, equivalent 2-fold partition could be made. Of the other five subjects, two gave an "indifference band" (i.e., chose wager (c) for $x = 800$ and $x = 850$), one had a "less than" choice among the "at least" choices (i.e., chose the "at least" bets for $x = 600$, 650, 750, 800, and 850, but the "less than"

bet for x = 700)*, and the other two subjects chose the same bet all nine times (one subject always chose the "at least" bet**, while the other always chose the (c), or indifference, bet).

Four of these 5 subjects recognized their inconsistent responses during the interview. They attributed their choices to carelessness and all four indicated a desire to modify their choices. The subject who always chose the indifference bet did not seem to understand the implications of his choice, and insisted that he would always choose the (c) wager--even if the values used in the experiment were replaced by much larger values. He stated that he had chosen the indifference wager "for consistency" and refused to discuss the experiment any further.

8. CONCLUSIONS

This study should be viewed as an exploratory one--an initial attempt to probe the normative implications of the decision theory postulates. Several characteristics of this study serve to distinguish it from others. The focus was on the level of the individual postulates rather than on the level of the expected utility theorem. The various counter-arguments to the postulates were examined in the context of controlled experiments. The subjects of these experiments were experienced, practicing decision makers, that is, high-level business executives.

---

* This is also an inadmissible choice--a violation of Postulate 3.

** This choice is not inconsistent if the subject thought that the value would be greater than 1000, but information obtained during the interview indicated that this was not so.

In addition to gathering evidence on the extent to which these decision makers conform to the theory in their actual decision behavior, a systematic followup was made on some normative implications of the postulates. In the process some preliminary indications of the teachability of the theory were obtained. The results pertaining to each of these points will be summarized below.

Not surprisingly, we find that the personal probability/utility theory is not a very good description of the decision making behavior of the subjects. From the previous sections of this paper, one can note a high degree of inconsistency between the actual choices of the subjects and those choices prescribed by the appropriate postulate. In addition, an examination of the responses over all experiments by each subject (see Chapter 10 of MacCrimmon [1965]) shows that the choices of each subject were inconsistent with at least one of the postulates. It follows logically then that no subject acted as if he maximized expected utility over all experiments.

In attempting to generalize this conclusion, two characteristics of the experiments must be considered. First, the decision problems were specially constructed in order to elicit violations of particular postulates. This suggests that the postulates might better describe more ordinary situations--that is, the less structured ones we would expect to confront in the real world. On the other hand, the decision problems were very simple and somewhat transparent and should make

conformity with the postulates much easier than in the more complex real world decision problems.

The results presented in this paper show that there was a high degree of apparent violation across most of the postulates in the actual written choices of the subjects. Later, when confronted with the written prepared answers, they tended to select those most similar to their own, although there was a slight trend toward the confroming answer. However, during the interviews, under a neutral form of questioning*, there were large shifts toward conforming with the postulates. In many cases the subjects themselves expressed a desire to change their choices even before the particular experiments were discussed. The violations were generally labelled as "mistakes" by the subjects and in one or two cases may be attributed to undue complexity or ambiguity in the experiments themselves. Thus, there was little persistent violation of the postulates. Most of the persistent violation that did occur involved Postulate 2. This tends to justify the interest in, and attention to, Postulate 2 (Savage's so-called "sure-thing principle") in the literature.

Most of the persistent violation, and also the highest degree of violation in the actual choices, occurred among the lower-level executives. The higher-level executives tended to have both a higher degree

---

* The interview results show that the subjects did not know the context of the experiments. It had been repeatedly emphasized that there was no right or wrong answer to the decision problems with which they were presented. For details on the exact form of the interview see MacCrimmon [1965].

of conformity in their actual choices and a better acceptance of the applicability of the postulate-based reasoning. The higher-level executives were less defensive in discussing their choices, and were able to draw on their own real world decision making experiences--a subject that was pursued in the latter part of the interviews. In addition, these executives (earning over $30,000/year) recognized the need for studies of decision making involving uncertainty, and were, in general, enthusiastic about the teachability of some of the decision theory concepts. This study should demonstrate that using business executives (especially high-level ones) as subjects in decision making experiments is not only feasible but highly desirable. The opportunities lost by using college students, or some similar group, in decision making studies should be recognized.

Perhaps the most important conclusion to be drawn from this study is the desirability of using the personal probability/utility postulates in training decision makers. As noted above, the most successful executives were especially enthusiastic about this possibility. Even though training was not an overt part of this study, that is, no pressures were applied to get the subjects to adopt the decision theory approach, we see that simple exposure, as described in this paper, led to a large shift toward acceptance of the postulates. More direct training methods can easily be constructed--some can be inferred from this paper. Future studies should systematically examine methods for training people how to structure decision problems and how to police their own decisions within the decision theory framework.

REFERENCES

Allais, M., "Fondements d'une Théorie Positive des Choix Comportant un Risque et Critique des Postulats et Axiomes de l'École Americaine," Econometrie, Paris: Centre National de la Recherche Scientifique, 1953.

Allais, M., "Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l'École Americaine," Econometrica, 21, 1953a, pp. 503-546.

Becker, G.M., M.H. DeGroot, and J. Marschak, "Measuring Utility by a Single-Response Sequential Method," Behavioral Science, 9, 1964, pp. 226-232.

Brim, O.G., and F.W. Koenig, "Two Aspects of Subjective Probability among College Students," J. Communication, 9, 1959, pp. 19-26.

Cohen, J., E.J. Dearnaley, and C.E.M. Hansel, "The Addition of Subjective Probabilities: the Summation of Estimates of Success and Failure," Acta Psychologia, 12, 1956, pp. 371-380.

Crandall, V.J., D. Solomon, and R. Kellaway, "Expectancy Statements and Decision Times as Functions of Objective Probabilities and Reinforcement Values," J. Personality, 24, 1955, pp. 192-203.

Davidson, D., J.C.C. McKinsey, and P. Suppes, "Outline of a Formal Theory of Value," Philosophy of Science, 22, 1955, pp. 140-160.

Davidson, D., P. Suppes, with S. Siegel, Decision Making: An Experimental Approach, Stanford, California: Stanford University Press, 1957.

Davidson, D., and J. Marschak, "Experimental Tests of a Stochastic Decision Theory" in Measurement: Definitions and Theories (edited by Churchman, C.W., and P. Ratoosh), New York: John Wiley and Sons, 1959.

Davis, J.M., "The Transitivity of Preferences," Behavioral Science, 3, 1958, pp. 26-33.

Edwards, W., "Probability Preferences in Gambling," Amer. J. of Psych., 66, 1953, pp. 349-364.

Edwards, W., "The Prediction of Decisions Among Bets," J. Exptl. Psychol., 50, 1955, pp. 201-214.

Ellsberg, D., "Risk, Ambiguity, and the Savage Axioms," Quar. J. Econ., 75, 1961, pp. 643-669.

Ellsberg, D., "Risk, Ambiguity, and the Savage Axioms," Quart. J. of Econ., 77, 1963, pp. 336-342.

Fellner, W., "Distortion of Subjective Probabilities as a Reaction to Uncertainty," Quart. J. of Econ., 75, 1961, pp. 670-689.

Herstein, I.N., and J. Milnor, "An Axiomatic Approach to Measurable Utility," Econometrica, 21, 1953, pp. 291-297.

Irwin, F.W., "Stated Expectations as Functions of Probability and Desirability of Outcomes," J. Personality, 21, 1953, pp. 329-335.

MacCrimmon, K.R., An Experimental Study of the Decision Making Behavior of Business Executives, unpublished dissertation, University of California, Los Angeles, 1965.

Marks, R.W., "The Effect of Probability, Desirability, and 'Privilege' on the Stated Expectations of Children," J. Personality, 19, 1951, pp. 332-351.

Marschak, J., "Rational Behavior, Uncertain Prospects and Measurable Utility," Econometrica, 28, 1950, pp. 111-141.

Marschak, J., "Actual Versus Consistent Decision Behavior," Behavioral Science, 9, 1964, pp. 103-110.

Marschak, J., and R. Radner, Theory of Teams, 1966, forthcoming.

May, K.O., "Transitivity, Utility and the Aggregation of Preference Patterns," Econometrica, 22, 1954, pp. 1-13.

McGlothlin, W.H., "Stability of Choices Among Uncertain Alternatives," Amer. J. Psychol., 69, 1956, pp. 56-67.

Morlat, G., "Comment on an Axiom of Savage," Econometrie, Paris: Centre National de la Recherche Scientifique, 1953, pp. 156-167.

Papandreou, A.G., "A Test of a Stochastic Theory of Choice," Univ. Calif. Publs. Econ., 16, 1957, pp. 1-18.

Pratt, J.W., H. Raiffa, and R. Schlaifer, "The Foundations of Decision Under Uncertainty: An Elementary Exposition," J. Am. Statist. Assoc., 59, 1964, pp. 353-375.

Raiffa, H., "Risk, Ambiguity, and the Savage Axioms: Comment," Quart. J. Econ., 75, 1961, pp. 690-694.

Ramsey, F.P., "Truth and Probability," The Foundations of Mathematics, London: Routledge and Kegan Paul, 1931.

Roberts, H.V., "Risk, Ambiguity, and the Savage Axioms: Comments," Quart. J. Econ., 77, 1963, pp. 327-335.

Rose, A.M., "A Study of Irrational Judgements," J. Polit. Econ., 65, 1957, pp. 394-403.

Savage, L.J., The Foundations of Statistics, New York: John Wiley and Sons, 1954.

von Neumann, J., and O. Morgenstern, Theory of Games and Economic Behavior (2nd ed.), Princeton, New Jersey: Princeton Univ. Press, 1947.

Unclassified

Security Classification

DOCUMENT CONTROL DATA · R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author)

Western Management Science Institute
University of California, Los Angeles
Los Angeles, California

2a REPORT SECURITY CLASSIFICATION

Unclassified

2b GROUP

3 REPORT TITLE

Descriptive and Normative Implications of the Decision Theory Postulates

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

Working Paper

5 AUTHOR(S) (Last name, first name, initial)

Kenneth R. MacCrimmon

6 REPORT DATE	7a TOTAL NO OF PAGES	7b NO OF REFS
March-April, 1966	31	33

8a CONTRACT OR GRANT NO	9a ORIGINATOR'S REPORT NUMBER(S)
Nonr 233(75)	Working Paper No. 102
b PROJECT NO: 047-041	
c	9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d	

10 AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited - Western Management Science Institute
University of California
Los Angeles, California 90024

11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY

13 ABSTRACT

This study deals with decision making under uncertainty, and more specifically with a set of postulates (based on the concepts of utility and personal probability) for consistent decision making. It explores the degree to which these postulates are good descriptions of actual behavior and the degree to which they seem to be good norms. Experiments in the form of decision problems based on each of the postulates were given to experienced business executives. In addition to obtaining basic responses to these problems, a systematic follow-up was made both in written and later in oral form in order to probe the normative implications of the postulates.

We conclude, based on the basic responses, that the decision making theory under study is not a very good description. However- and this is the important point- the subjects tend to regard most of the deviations from the theory as mistakes and if given the opportunity will correct them. This suggests the importance of the postulates in training decision makers in both how to structure decision problems and also how to police their own decisions.

DD FORM 1 JAN 64 1473 0101-807-6800

Security Classification